Popper's Philosophy and Learning

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Abstract

This short article introduces the connection between Popper's philosophy on scientific discoveries and Bayesian learning.

1 Popper

Sir Karl Raimund Popper (1902−1994), was an Austrian-born British philosopher of science. He was educated at the University of Vienna. His first book, *Logik der Forschung* (*The Logic of Scientific Discovery*) (Popper 1959) has become a classics in philosophy of science, in which he criticized psychologism, naturalism, inductionism, and logical positivism, and put forth his theory of potential falsifiability being the criterion for what should be considered science.

Popper argued strongly against the observationalist-inductivist account of science. That makes him agreed with Hume on the Problem of Induction. He also pointed out the importance of falsification as the demarcation of science and non-science and the asymmetry of falsification and verification. His points of views are actually supporting Bayesian probability rather than frequentists' view towards probability. His belief in Evolutionary Epistemology may guide the future direction of the research of learning and inductive reasoning.

He is counted among the most influential philosophers of science of the 20th century, and also wrote extensively on social and political philosophy. Popper is perhaps best known for repudiating the classical observationalist-inductivist account of science; by advancing empirical falsifiability as the criterion for distinguishing scientific theory from non-science; and for his vigorous defense of liberal democracy and the principles of social criticism which he took to make the flourishing of the "open society" possible. In his *The Open Society and Its Enemies*, he wrote:

> Man has created new worlds − of language, of music, of poetry, of science; and the most important of these is the world of the moral demands, for equality, for freedom, and for helping the weak.

Here in this article we are not trying to discuss his philosophy on social science and humanity. We will only discuss his philosophy in scientific discovery.

2 Problem of Induction

The method of basing general statements on accumulated observations of specific instances is know as *induction*[1], and it seen as the criterion of demarcation between science and non-science. Scientific

[1] This section is basically based on the second chapter of (Magee 1973)

statements are based on observational and experimental evidence, contrasted with statements of all other kinds, whether based on authority, or emotion, or tradition, or speculation, or prejudice, or habit, or any other foundations, that are referred to as non-science.

Hume pointed out that no number of singular observation statements, however large, could logically entail an unrestrictedly general statement. For example, we observe that sun rises every morning, and we conclude that sun *will* rise tomorrow morning. However, but it is not logically entailment of our previous knowledge. In another words, scientific laws have been found to hold good in the past does not logically entail that they will continue to hold good in future. Also, the whole of our science assumes that regularities of nature - assumes that the future will be like the past in all those respects in which natural laws are taken to operate - yet there is no way in which this assumption can be secured[2]. This is often referred to as the problem of induction or Hume's problem.

According to geometry, the sum of the three angles of an arbitrary triangle is 180°, that is 100% true. However, since you cannot draw all the triangles, probably there is a special triangle which is not fit for this rule. So, it is not absolutely true when considering the problem of induction. Is it a paradox? The main difference from the induction of given evidence and such proof involving 'arbitrary' facts is described as follows: If you are given a triangle, we can prove that the sum of its angles is 180°. Or in another words, given an existence of a triangle, it is satisfying, that is *independent* from previous observations. However, about tomorrow's sunrise, we cannot prove it in the same way, we can only predict it based on previous experience (see Appendix I).

3 Popper and Bayesianism

The essence of the Bayesian approach is to provide a mathematical rule explaining how can we update existing beliefs in the light of new evidence. In order to formulate a theory, we need to combine new observations with the existing knowledge or expertise. This rule gives an insight how can we generalize observations to theories.

Popper's view on learning and induction can be regarded as philosophy of Bayesianism in modern age. What we have learn from observations or data are not logical reasoning but probability inference. We just simply pick one which explains observations better according to our prior of theories. If it is our mind that invented theories. The prior then is the prior of theories in our psychological minds.

To prove the truth of a theory, we justify the belief in a theory, since this is to attempt is logically impossible. Say, given an observation, a statistician likes to explain it by statistics, a physicist prefer physics, chemistist prefer chemistry and so on, all of them are not logical reasoning but belief updating based on the prior of their psychological minds. All theories could be right[3] because they may all explain the given observation. However, all of them could be wrong because of unjustified priors. In a logical world, a truth will result a truth, this law does not work properly in learning. So that Popper argues that pure logical induction does not exist at all! Because mathematical inference or learning are not logical consequence of given conditions.

Give a set of data, the ways of generating these data could be numerous. How can we expect to find a hypothesis to explain the data. If we call the space, that contains all the hypotheses that are consistent with given data, version space. What we often do is select one of the hypothesis from the version space according to some criteria (e.g. Occam's razor). Such that the hypothesis we 'generalize' from the given data is NOT logical entailment of the given data. This generalization is actually 'deduced' from the given data and some prior knowledge of how to choose a hypothesis from the version space. This deduction is actually probability inference. More precisely, it is Bayesian

[2]For example, we assume the law of physics is independent to time and locations. There could be the possibility that time itself is playing a role in some experiments. At least, we cannot prove that it is not the case.

[3]We assume that different actions may have the same consequences. This is also one of the reasons that induction is not reliable as deduction, because we usually believe that same actions have same results.

rule. To update our prior knowledge based on given data.

This article may become very fat if we consider the philosophy of Bayesian learning. If you are interested in such topics, you can read (Jaynes 2003).

4 Evolutionary Epistemology

Evolutionary epistemology is a theory in metaphysics, applying the concepts of biological evolution to the growth of human knowledge and, in particular, scientific theories. If we observe the our history of science and technology, all theories are *temporarily* right. From Newton to Einstein, human knowledge on the physical world is gained by trial and error. For the evolutionary epistemologist, all theories are true only provisionally, regardless of the degree of empirical testing they have survived. As such, it bears remarkable similarities to the process of evolution by natural selection.

One of the hallmarks of evolutionary epistemology is the notion that empirical testing does not justify the truth of scientific theories, but rather that social or methodological processes select those theories with the closest "fit" to a given problem better under some limitations. As the science advances and those limitations do not exist any longer, new theories will be found to fit to the problems better than old theories. The fitness of a theory, then, may be related to the concept of biological fit; that, although adaptation is the process by which a species survives the hazards of its environment, fitness in the present does not predict continued survival.

Let us finish this article with a quote from Popper on knowledge and ignorance.

> The more we learn about the world, and the deeper our learning, the more conscious, specific, and articulate will be our knowledge of what we do not know, our knowledge of our ignorance. For this, indeed, is the main source of our ignorance–the fact that our knowledge can be only finite, while our ignorance must necessarily be infinite.

Appendix I - Rule of Succession

Suppose $X_1, \cdots, X_N, X_{N+1}$ are conditionally independent random variables given the value of p. $P(X_i|p)$ is a Bernoulli distribution (i.e. X_i is either 0 or 1 for $i = 1, \cdots, N+1$).

$$P(X_i|p) = p^{X_i}(1-p)^{1-X_i}$$

where p is uniform distributed in $[0,1]$ (i.e. $p \sim \mathcal{U}[0,1]$). Given previous N trails, there are S trails being true (i.e., $X_i = 1$). We hope to calculate the probability that the next state is also true. That is: $P(X_{N+1} = 1|X_1+, \cdots, +X_N = S)$. This succession model only has one parameter p. Therefore, this probability can be consider as the estimation of p given a series of data. Likelihood of p given evidence is:

$$L(p) = L(X_1, \cdots, X_N|p) = \prod_{i=1}^{N} P(X_i|p) = p^S(1-p)^{N-S} \tag{1}$$

Because $p \sim \mathcal{U}[0,1]$, according to Bayes rule, the posterior probability:

$$f(p) = P(p|X_1, \cdots, X_n) = \frac{L(p)}{\int L(p)} \tag{2}$$

The integral is:

$$\int L(p) = \int_0^1 p^S(1-p)^{N-S} = \frac{S!(N-S)!}{(N+1)!}$$

Therefore, by combining eq. 1 and 2:

$$f(p) = \frac{(N+1)!}{S!(N-S)!} p^S(1-p)^{N-S} \tag{3}$$

Finally (Online Source 2007),

$$E[f(p)] = \frac{S+1}{N+2} \tag{4}$$

In frequentists' approach, $f(p) = \frac{S}{N}$.

References

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