Impact of Social Network Structure on Social Welfare and Inequality

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Abstract. In this chapter, how the structure of a network can affect the social welfare and inequality (measured by the Gini coefficient) are investigated based on a graphical game model which is referred to as the Networked Resource Game (NRG). For the network structure, the Erdos-Renyi model, the preferential attachment model, and several other network structure models are implemented and compared to study how these models can effect the game dynamics. We also propose an algorithm for finding the bilateral coalition-proof equilibria because Nash equilibria do not lead to reasonable outcomes in this case. In economics, increasing inequalities and poverty can be sometimes interpreted as a circular cumulative causations, such positive feedback is also considered by us and a modified version of the NRG by considering the positive feedback (p-NRG) is proposed. The influence of network structures in this new model is also discussed at the end of this chapter.

Keywords: Networked Resource Game; P-NRG Network Formation; Graphical Games; Nash Equilibrium

1 Introduction

Modern game theory began with the idea regarding the existence of mixed-strategy equilibria in two-person zero-sum games and its proof by John von Neumann. His book *Theory of Games and Economic Behavior* [27] published in 1944 considered cooperative games of several players. The second edition of this book provided an axiomatic theory of expected utility, which allowed mathematical statisticians and economists to treat decision-making under uncertainty. His later paper published in 1952 introduced a polynomial algorithm, it is the first work on algorithm even before the appearance of digital computer. The modern game-theoretic concept of *Nash Equilibrium* is defined in terms of mixed strategies, where players choose a probability distribution over possible actions. The contribution of John Forbes Nash in his 1951 paper *Non-Cooperative Games* was to define a mixed strategy Nash Equilibrium for any game with a finite set of actions and prove that at least one (mixed strategy) Nash Equilibrium must exist in such a game. How to develop efficient algorithms for calculating the Nash Equilibrium in a given system is a focus of modern *Algorithmic Game Theory* [21].

In recent years, graphical games have attracted much attentions for modeling social phenomena. This emerging research provides new approaches to investigate problems such as group consensus making, networked bargaining and trading strategies. In this chapter, we introduce the *Networked Resource Game* (NRG) to investigate the interactions of a society where actions are resource-bounded, i.e., agents have limits on how they are able to act across their network. In this model, agents have a finite number of resources and their network structure may affect how those resources can be coupled with others' resources in order to produce social rewards. One example of this is in professional networks where agents need to form partnerships and the payoffs of the partnerships are determined by a function of their capabilities.

Few work has been reported to study the network structure and its dynamics affect social welfare and inequality, measured by the Gini coefficient [7], of the resulting equilibria. For the network structure, we utilize the *Erdos-Renyi* (ER) model [23], the *Preferential Attachment* (PA) model [1], and several other structure models. We propose an algorithm to find bilateral coalition-proof equilibria because Nash equilibria do not lead to reasonable outcomes in this case. In previous research [18], preliminary results have been obtained to show the impact of network structures on game dynamics. In this chapter, more comprehensive results are presented .

In the NRG model, we only consider the cooperations between agents through bilateral resource consumption to obtain the reward. However, it is not a very good assumption considering the real-world cases where cooperations and competitions are co-existing. In this chapter, we present a new NRG model with positive feed-back (p-NRG) to simulate both cooperation and competition scenarios in a game. For both the NRG and p-NRG models, we study the impact of network structures on the game dynamics in terms of resource allocation, social welfare and inequality.

The remaining of this chapter is structured as the following. Section 2 gives a full review on related works from game theory to graphical game models. Section 3 introduces the networked resource game in details. In Section 4, we introduce a set of network structure models for empirical evaluations presented in Section 5. In Section 6, we introduce the NRG model with positive feedback and empirical evaluations are also given. Finally, the conclusions and future work are given in the last section.

2 Related Works

2.1 Game Theory and Intelligent Decision Making

Game theory has influenced many research fields including economics (historically its initial focus), political science, biology, and many other fields. In recent years, its presence in computer science has become impossible to ignore. It features routinely in the leading conferences and journals of artificial intelligence (AI), theory, certainly electronic commerce, as well as in networking and other areas of computer science. According to Shoham [25]:

> One reason for such binding is application pull: the Internet calls for analysis and design of systems that span multiple entities, each with its own information and interests. Game theory, for all its limitations, is by far the most developed theory of such interactions. Another reason is technology push: the mathematics and scientific mind-set of game theory are similar to those that characterize many computer scientists.

Increasing requirements of e-commerce initiate the development of this interdisciplinary research field. Especially in the field like market mechanism designs, AI theory such as machine learning [29] and evolutionary computing [28] had played important roles. Game theory is a framework to explore the interaction among self-interested players. It can be explained as the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. An alternative term suggested it as an interactive decision theory. Such decision making is inseparable to so called multiple interacting intelligent agents within an environment.

An *Intelligent Agent* (IA) is generally regarded as an autonomous entity which observes through sensors and acts upon an environment using actuators. Intelligent agents may also learn or use knowledge to achieve their goals. Though an intelligent agent may have a physical structure (e.g., an autonomous robot), it is generally a software entity that carries out some set of operations on behalf of a user or another program with some degree of independence, and in so doing, employ some knowledge or representation of the user's goals or desires. In game theory, agents can be used for experimental studies. Agents are modeled to have bounded rationality for some decision making. In this research, homogeneous agents are used in graphical games.

2.2 Graphical Games

In social and economic interactions - including public goods provision, job search, political alliances, trade, partnership, and information collection - an agent's well being depends on his or her own actions as well as on the actions taken by his or her neighbors. Such neighboring relations can form a network whose structure decides the direct interaction. The literature identifying the effects of agents' neighborhood patterns (i.e., their social network) on behavior and outcomes has grown over the past several decades. The emerging empirical evidence motivates the theoretical study of network effects. We would like to understand how the pattern of social connections shape the choices that individuals make and the payoffs they can hope to earn.

In this research, we mainly focus on the network structure in order to understand how the changes in network structure will reshape the game. In recent years, the games played on networks have been studied [14, 15]. A general framework for the study of games in such an incomplete-information setup has been developed [13]. Graphical games [16] are a representation of multiplayer games meant to capture and exploit locality or sparsity of direct influences. They are most appropriate for large population games in which the payoffs of each player are determined by the actions of only a small subpopulation. As such, they form a natural counterpart to earlier parametric models. Whereas congestion games[1] and related models implicitly assume a large number of weak influences on each player, graphical games are suitable when there is a small number of strong influences.

Generally, a graphical game can be described at the first level by an undirected graph G in which players are identified with vertices. The semantics of the graph is that a player or vertex i has payoffs that are entirely specified by the actions of i and those of its neighbor set in G. Thus G alone may already specify strong qualitative constraints or structure over the direct strategic influences in the game. To fully describe a graphical game, we must additionally specify the numerical payoff functions to each player but now the payoff to player i is a function only of the actions of i and its neighbors, rather than the actions of the entire population. In the many natural settings where such local neighborhoods are much smaller than the overall population size, the benefits of this parametric specification over the normal form are already considerable.

2.3 Nash Equilibria for Graphical Games

It is known that finding Nash equilibria for graphical games is difficult even for restricted structures [5]. Local heuristic techniques are commonly employed [4, 10]. A seminal work in using agent-based simulation to study human interaction was Axelrod's tournament for Prisoner's Dilemma [2]. Prisoner's Dilemma has also been studied in a graphical setting with simulated agents [20]. Dynamic networked games based on the Ultimatum Game have also been investigated [17] Research on identification and development of networks includes analyzing event-driven growth [24] and inferring social situations by interaction geometry [9].

Some other works have described algorithmic methods to discover temporal patterns in networked interaction data [11]. Researchers have formulated efficient solution methods for games with special structures, such as limited degree of interactions between players linked in a network, or limited influence of their action choices on overall payoffs for all players [16, 22, 26]. Another line of research focuses on the design of agents that must maximize their payoffs in a multi-player setting. If self-play convergence to Nash equilibrium is a desiderata for agent policies, In [3], the authors show the convergence of certain kinds of policies in small repeated matrix games. If correlated Nash equilibrium is our goal, it has also been shown that using another set of adaptive rules will result in convergence to a correlated equilibrium [8]. Other work has taken a

[1] http://en.wikipedia.org/wiki/Congestion_game

different approach, and does not presume the equilibrium is a goal; rather profit maximization is the only metric.

Part of the original motivation for graphical games came from earlier models familiar to the machine learning, AI and statistics communities collectively known as graphical models for probabilistic inference, which include Bayesian networks, Markov networks, and their variants. Both graphical models for inference and graphical games represent complex interactions between a large number of variables (random variables in one case, the actions of players in a game in the other) by a graph combined with numerical specification of the interaction details. In probabilistic inference the interactions are stochastic, whereas in graphical games they are strategic (best response). The connections to probabilistic inference have led to a number of algorithmic and representational benefits for graphical games.

Graphical games adopt a simple graph-theoretic model. An n-player game is given by an undirected graph on n vertices and a set of n matrices. The interpretation is that the payoff to player i is determined entirely by the actions of player i and his neighbors in the graph, and thus the payoff matrix for player i is indexed only by these players. We thus view the global n-player game as being composed of interacting local games, each involving (perhaps many) fewer players. Each player's action may have global impact, but it occurs through the propagation of local influences. Formally, a graphical game model is a tuple

$$[I, \{A_i\}, \{J_i\}, \{u_i(\cdot)\}]$$

where I and A_i are as before, J_i is a collection of players connected to i, and $u_i(a_i, a_{J_i})$ is the payoff to player i playing $a_i \in A_i$ when the players J_i jointly play a_{J_i}. The notation $a_{J_i} \subset a_{-i}$ means that each $j \in J_i$ plays its assigned strategy from a_{-i}. We define A_{J_i} to be the set of joint strategies of players in i's neighborhood. This game structure is captured by a graph

$$G = [V, E] \tag{1}$$

in which $V = I$ (i.e., each node corresponds to a player) and there is an edge

$$e = (i, j) \in E \quad if \quad j \in J_i$$

The graph topology might model the physical distribution and interactions of agents: each sales-person is viewed as being involved in a local competition (game) with the salespeople in geographically neighboring regions. The graph may be used to represent organizational structure: low-level employees are engaged in a game with their immediate supervisors, who in turn are engaged in a game involving their direct reports and their own managers, and so on up to the CEO. The graph may coincide with the topology of a computer network, with each machine negotiating with its neighbors (to balance load, for instance).

Graphical games also provide a powerful framework in which to examine the relationships between the network structure and strategic outcomes. Of particular interest is whether and when the local interactions specified by the graph

G alone (i.e., the topology of G, regardless of the numerical specifications of the payoffs) imply nontrivial structural properties of equilibria. It can be shown that different stochastic models of network formation can result in radically different price equilibrium properties. In [21], the authors give an example that considers the simplified setting in which the graph G is a bipartite graph between two types of parties, buyers and sellers . Buyers have an endowment of 1 unit of an abstract good called cash, but have utility only for wheat; sellers have an endowment of 1 unit of wheat but utility only for cash. Thus the only source of asymmetry in the economy is the structure of G. If G is a random bipartite graph (i.e., generated via a bipartite generalization of the classical Erdos-Renyi model), then as n becomes large there will be essentially no price variation at equilibrium (as measured, for instance, by the ratio of the highest to lowest prices for wheat over the entire graph). Thus random graphs behave "almost" like the fully connected case. In contrast, if G is generated according to a stochastic process such as preferential attachment. the price variation at equilibrium is unbounded, growing as a root of the economy size n.

In previous work, there has been tremendous interest in agent strategies in different games and auctions. Many have shown to optimize the social welfare, reduce the inequality and reach equilibrium in the social network [19], or how to get stable in a network, where the participants have special preference [12]. Gini coefficient is commonly used as a measure of inequality of income or wealth. Its value depends on both income inequality and other factors such as the network structure [7, 30].

3 Networked Resource Game Model

Similar to a standard graphical game, the Networked Resource Game is defined by a set of N players $\{p_i\}_{i=1}^N$, a card distribution C, a graph G and a reward function R. At each round, the games are played based on a matrix M where each element indicating a link exists between the two players. In the Networked Resource Game, the actions are based on available resources, which we will informally call *cards*. Each player p_i has a set of cards:

$$C_i = \{c_{i,1}, \dots, c_{i,N_i^C}\}$$

where N_i^C is the number of cards for the player p_i. The cards represent a skill or resource that the player can play on a link. The graph specifies the links over which players may play their cards. Here, we include the restriction that a player may play at most one card on a link. Thus, the number of cards indicate a players ability to have multiple simultaneous partnerships. It is possible that a player has more links than cards and also more cards than links.

Each card has a type which comes from a predetermined type set T, i.e., $c_{i,j} \in T$. For simplicity, given a discrete type set, we can think of the type as a color and that each card has a particular color. The graph $G = \{e_{ij}\}$ which has undirected edges between the players, where $e(i, j)$ denotes the link between

p_i and p_j. It is possible that some players have no links between them. Each card can only be used once per round. Each card is one of T types, and the matrix game can be defined by a $(|T|+1) \times (|T|+1)$ payoff matrix where each row/column is one of the $|T|$ card types or the null action, and where the value pairs in the cells are the rewards for the two players.

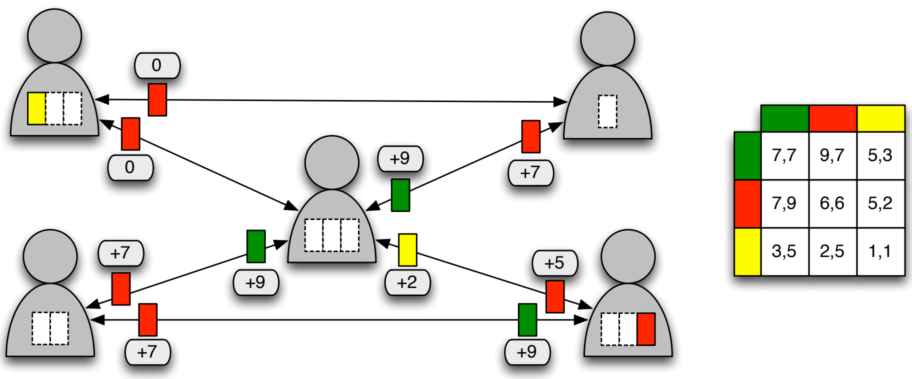

Fig. 1. An example of the Networked Resource Game. The matrix on the right-hand side gives the payoff matrix by playing different type of cards. Each agent is initialized with different type and number of cards. For example, the player at the center of the network has 3 cards and they are 2 greens and one yellow. By playing these cards with 3 of his 4 linked neighbors he received rewards of $9+9+2 = 20$ that is calculated based on the given payoff matrix. For the agent on the upper left corner, he has received 0 payoff because there is no card-playing between him and his two linked neighbors.

As usual, in this chapter, we assume that M is fixed, is symmetric, and is the same for every link. Based on what cards are played on a link, each player gets a reward specified by the function $R(a, \bar{a})$ and $a, \bar{a} \in A$ where A is the action space. The payoff matrix R (we use the same R for the reward function as well) is defined by:

$$R(a, \bar{a}) \qquad a, \bar{a} \in A$$

For example in Fig. 1, the payoff matrix is shown:

$$R(green, green) = (7, 7)$$

$$R(green, red) = (9, 7)$$

and so on. However, it implicitly contains the situation of null actions like the following:

$$R(null, red) = (0, 0)$$

So that the acutual action space for player p_i on link e_{ij} is

$$A_{ij} = C_i \cup 0 \tag{2}$$

where 0 indicates that the player chose not to play one of their cards on that link. Similarly, for the player j its action space is:

$$A_{ji} = C_j \cup 0 \tag{3}$$

The reward function R has $(|T|+1)^2$ inputs representing every combination of actions, i.e., all card types and not playing a card, for each player. The Networked Resource Game is similar to a standard graphical game, however, the action space has restrictions over multiple links whereas in standard graphical games, actions on link are independent. Here, we have the restriction that $\cup_j a_{ij} \subset C_i$ where a_{ij} is player p_i's action on link e_{ij}. This states that a player cannot play more cards than they have, which introduces a coupling over links.

An illustration of the game is shown in Fig. 1. It shows a game involving three card types (*green*, *red* and *yellow*). One can imagine that these cards represent assets of value in an economy that yield different outcomes to each contributor in partnerships. For example, a green card could represent capital, red could represent skilled labor and yellow could represent unskilled labor. Different combination of these resources may result in different rewards. For example, capital plus skilled labor may yield much more rewards than capital plus unskilled labor for both sides.

4 Network Formation and Finding Equilibria

4.1 Network Structure Models

In this section we describe a few models we use to create social network graphs and how to find the equilibria for a given graph is discussed. Network formation is determined by various growth processes that describe how a link is added to an existing graph. In this chapter, we use the following four network structure models:

Erdos-Renyi (ER)

The Erdos-Renyi (ER) model is either of two closely related models for generating random graphs. In the $ER(n, M)$ model, where n is the number of nodes in a graph. A graph is chosen uniformly at random from the collection of all graphs which have n nodes and M edges. For example, in the $G(3, 2)$ model, each of the three possible graphs on three vertices and two edges are included with probability 1/3.

In the $ER(n, p)$ model, a graph is constructed by connecting nodes randomly with probability p independent from every other edge. This model is named after Paul Erdo and Alfred Renyi who published *On the Evolution of Random Graphs* in 1960 [6]. This is a baseline process where we add a link chosen uniformly from those links that do not already exist in the graph. In this model, we connect each pair of nodes with some given probability (See Fig. 2).

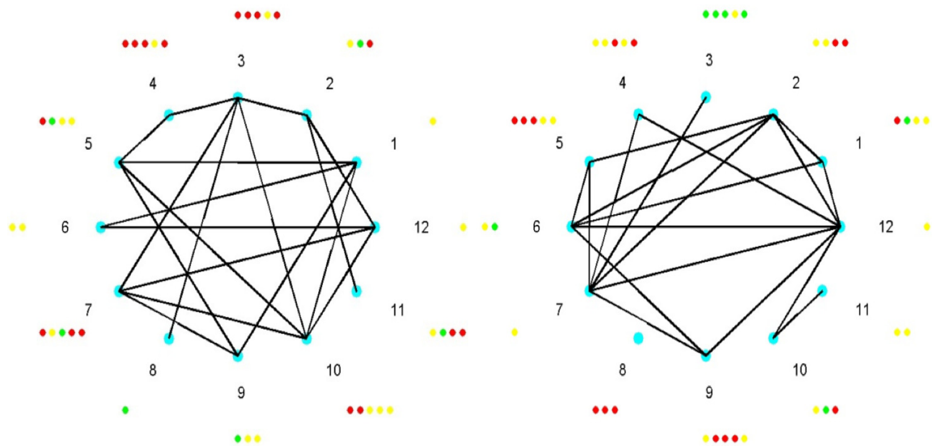

Fig. 2. An example of the ER model (left) and the PA model (right), where the red, yellow, and green dots represent the randomly distributed cards for each of the 12 players.

Preferential Attachment (PA)

If the input graph has zero or one link, we use the ER process. Thus, the network is seeded with two random links. After this, in order to add a link, we choose a node randomly and consider the links it could add to the graph, i.e., the set of links connected to the chosen node that are not already in the graph. Each such link is given a weight equal to the degree of the target node it connects to, and a link is chosen in proportion to these weights. Preferential attachment models have been proposed as a model that reflects how social networks are formed, particularly online.

Most Free Cards (MFC)

Each node is given an MFC score: the number of cards it has minus the number of links it has, i.e., a measure of the number of free cards for that player. The process selects a node uniformly from those that have the highest MFC score. This node then chooses a link uniformly from other nodes that have the highest MFC score. When the MFC scores are all zero, the algorithm becomes ER.

Poor-to-Rich Chain (PRC)

We first associate each player with a wealth calculated as the sum of the value of their cards, where the value of each card is the maximum reward obtainable from applying that card:

$$w_i = \sum_{c \in C_i} \max_{a \in T \cup 0} R(c, a) \tag{4}$$

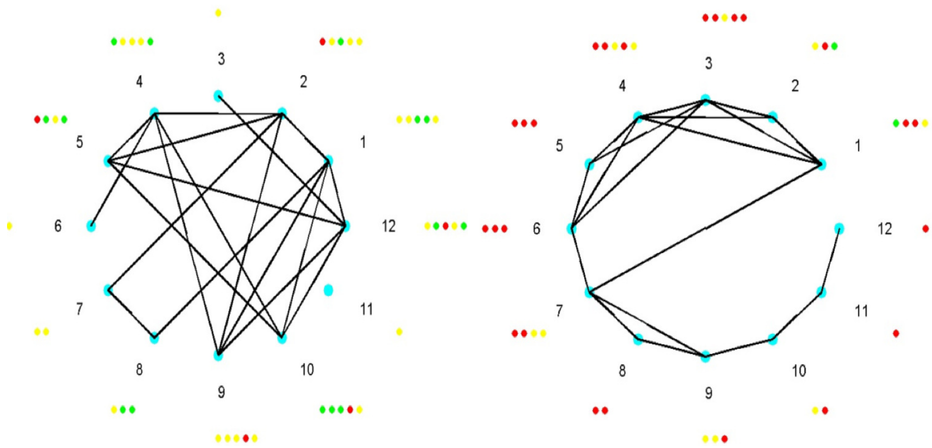

Fig. 3. An example of the MFC model (left) and the PRC model (right), where the red, yellow, and green dots represent the randomly distributed cards for each of the 12 players.

We first create a chain, where agents are ordered by wealth with ties broken randomly. Then, a player is chosen uniformly from those with the highest MFC score adds a link. The target node is the closest node in the chain with a free card, i.e., an MFC score greater than zero.

The various processes described above capture various degrees of control that players may have over the network on which they play. In the ER and PA models, players have no control over links. One may consider PA as player driven, but the game properties (card, rewards) do not affect the formation of the links so the processes are not strategic. The MFC model is a decentralized strategic model where agents have partial information about the state of the world, namely the number of cards and links for each player. The PRC model is a centralized model that takes game parameters into account when making the graph and incorporates a social structure into the world where people with similar wealth are more likely to be connected to each other.

4.2 Finding Equilibria

Given a game structure (cards, rewards, and a graph), we would like to determine an appropriate outcome. Nash equilibria are often considered as a solution for graphical games, however, it has some issues for the Networked Resource Game. Consider a simple example of four players in a sequentially connected graph where each player has only one card. Two players have a single red card and two players have a single green card. Let the rewards for having two cards with same color on a link be 100 points of reward for each player, two cards with different colors on the same link be 10 points, and all links with one or zero links be worth nothing (see Fig. 4).

Consider the situation where we have two red-green links and each player receives 10 points of reward. For that case, each player has no incentive to deviate, i.e, move their one card to another link, because that would cause a loss of 10 points, even though each player has a link to a player with the same color card. Thus, in the Networked Resource Game, Nash equilibria lead to artificially poorer results than one would expect if one was playing this game assuming players could communicate over the links that they have. Thus, we consider equilibria where players can make bilateral deviations. An equilibrium in this context is a state where no player would choose to make a unilateral deviation and no two players would choose to make a bilateral deviation. We use the procedure below to discover such equilibria for a given game structure.

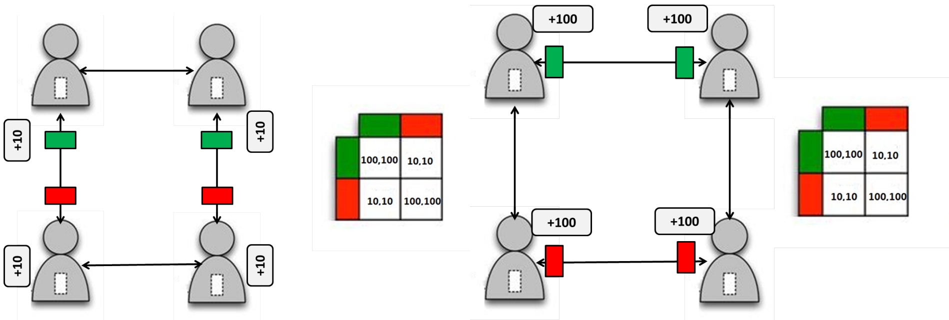

Fig. 4. Nash equilibria may not yield the best rewards for the Networked Resource Game. Considering the case that 4 players are connected sequentially. In the left-hand side figure, the game is with the Nash equilibria as none of the players has motivation to make unilateral change that will result in losing 10 point rewards though they may obtain much more points (+100) by making bilateral change as shown in the right-hand side figure.

Each player first assigns cards randomly to available links. We then perform action updates in a series of rounds. In each round, we order the set of links. For each link, the players iterate back and forth on card choices for the link. On the first iteration, the first player assumes that the other player plays one of their cards, chosen from all cards that player has, i.e., not necessarily the card being played on the link currently. The first player then plays their best response on all links given the cards that are played on all the links that they have. In the second iteration and all following iterations, the acting player chooses their best response to the cards that are being played on their link. This procedure continues until an equilibrium is reached for that link or we reach a preset limit of interactions. We continue this procedure for all links in each round. The procedure terminates, when at the end of a round, the joint actions are the same as the joint actions in the previous round. The procedure continues for a preset number of rounds. Finding equilibria in graphical games is a challenging problem. The algorithm presented at below is sound in that if it terminates before reaching the preset number of rounds, we know that the resulting joint action is an equilibrium for the game, however, we may not find all equilibria.

Pseudo Code of FINDING-EQUILIBRIA for computing bilateral coalition-proof equilibria

```
Algorithm FINDING-EQUILIBRIA
Inputs: one game structure(cards,rewards and a graph)
Outputs: bilateral coalition-proof equilibria
  Each player first assigns cards randomly to available links
  equilibria ← ∅
  for round ← 1 to n1
    do order the set of links
    num ← 1
    repeat
      (1)one player P_i assumes that the other player P_j
         which links with P_i plays one of its cards
      (2)P_i plays their best response on all links given the
         cards that are played on all the links that they have
      (3)P_j chooses their best response to the cards
         that are being played on their link
      (4)num ← num + 1
    until an equilibrium is reached for that link
          or num = n2
    if the joint actions = joint actions in the previous round
    then return equilibria
    else return -1
```

5 Experimental Studies

The Gini coefficient (also known as the Gini index or Gini ratio) [7, 30] is a measure of statistical dispersion, it is named after the Italian sociologist Corrado Gini[2]. It measures the ratio of areas above the Lorenz curve which plots the proportion of the total income of the population that is cumulatively earned by the bottom $x\%$ of the population. The Gini coefficient measures the inequality among values of a frequency distribution. A Gini coefficient of zero expresses perfect equality, while the Gini coefficient of one expresses maximal inequality among values (for example where only one person has all the income in a society), i.e., larger Gini coefficients indicate greater income disparity. In this chapter, the Gini coefficient is used as a measure of inequality.

In our experiments, we considered a society of consisting 12 players. In each round, each player was given a number of cards chosen uniformly from one to five:

$$|C_i| \sim U(1, 5)$$

[2] http://en.wikipedia.org/wiki/Gini_coefficient

We had three card types: *green*, *red*, and *yellow*. Card colors were selected independently for each card using the following probabilities:

$$P([\text{green red yellow}]) = [0.20\ 0.40\ 0.40]$$

5.1 Reward Functions

There were two methods for selecting reward functions to generate the payoff matrix: (1) In the baseline method, each reward for links with two cards on them were chosen randomly:

$$R(c_1, c_2) \sim U(1, 1000) \quad for\ c_1, c_2 \in T \tag{5}$$

Links with one or zero cards gave zero reward to both players. (2) In the alternate method, the reward for an arbitarily chosen link (e.g. green-green) is replaced with 100 times the value of the maximum of all the rewards in the baseline method. The latter is to investigate a society where there is a significantly outlying reward available to a small number of people if they make the right connections. We can exaggerate the variance of rewards in such a way we can observe the game dynamics more clearly.

It is for this reason that the green cards occur at lower likelihood than the others. For a given game card and reward structure, we would run our various network formation algorithms and generate graphs of increasing size. Each network formation algorithm was run 10 times, thus generating 10 graphs with the same number of edges for each process. For each game structure (cards, rewards and graph) that resulted, we would find the set of equilibria. For each graph, the equilibrium-finding algorithm was run 40 times and each run was ended if the algorithm didn't terminate in 15 rounds.

For any single equilibrium, we calculated the social welfare as the sum of all the rewards to all players and the Gini coefficient. For each game, we calculated an associated social welfare with the weighted average of social welfares of equilibria of that game, where weights were the number of times the equilibrium was discovered. We calculated associated Gini coefficients for each game structure similarly.

The Gini coefficient is normalized between zero (everyone has equal wealth) and one (one person has all the wealth), but social welfare for each game is a function of the payoff matrix. We use the following way to normalize the social welfare. First, we need to calculate the maximum value of all possibly generated social welfare by:

$$\max \sum_{(c_1,c_2)\in C_2} n_{c_1,c_2} \left(R(c_1,c_2) + R(c_2,c_1)\right) \tag{6}$$

$$\text{such that} \sum_{\tilde{c}\in T} n_{c,\tilde{c}} \leq n_c \quad \forall c \in T, \quad n_{c,\tilde{c}} \geq 0, \quad \forall c, \tilde{c} \tag{7}$$

This considers all possible combinations of cards on a link $(c_1, c_2) \in C_2$ and maximizes the reward obtained for having a particular number of card combinations on the graph (n_{c_1,c_2}) with the rewards obtained for that card combination

$(R(c_1, c_2) + R(c_2, c_1))$, such that the number of card combinations of the graph does not violate the card constraints, i.e., the number of cards of a particular type (n_c) and non-negativity of the number of combinations. This yields an upper bound on the social welfare because it allows multiple links between players and links between cards of the same player. We use this to normalize social welfares across different card and reward structures.

5.2 Experimental Results

Fig. 5 shows how social welfare changes as a function of network formation algorithm and graph size. We did not show the error bars for clarity in presentation but we discuss significance below. We see that social welfare improves as the society gets more connected for all algorithms. MFC and PRC are significantly better than ER and PA. ER is slightly better than PA but the result is not statistically significant. These results hold in both reward scenarios. For baseline rewards, MFC and PRC both reach about 0.9 efficiency in social welfare at about 18 links and do not improve much beyond that. We also see the impact of network structure as the 28-link ER and PA graphs are less efficient that MFC and PRC graphs that are half the size. For alternate rewards, the efficiency is significantly smaller than the baseline word, this could be the result of two factors: there are green-green links that are not being formed, and our normalization could be overcounting the number of potential green-green links.

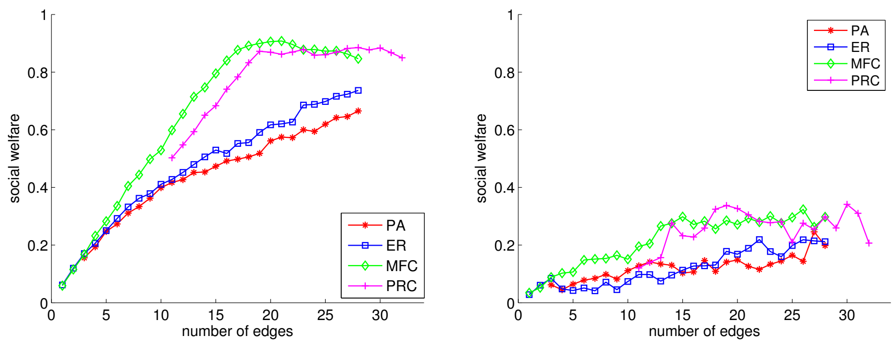

Fig. 5. Social welfare of given structure models with increasing number of edges. The left-hand side figure is with baseline rewards and the right-hand side one is with alternate rewards.

Fig. 6 shows how Gini coefficients change as a function of network formation algorithm and graph size. Inequality decreases as the network sizes increase. For the baseline reward structure, MFC, PRC and ER are significantly better than PA. The key change is that ER has jumped from the PA equivalence class to the MFC/PRC equivalence class. We note that the Gini coefficient is relatively flat

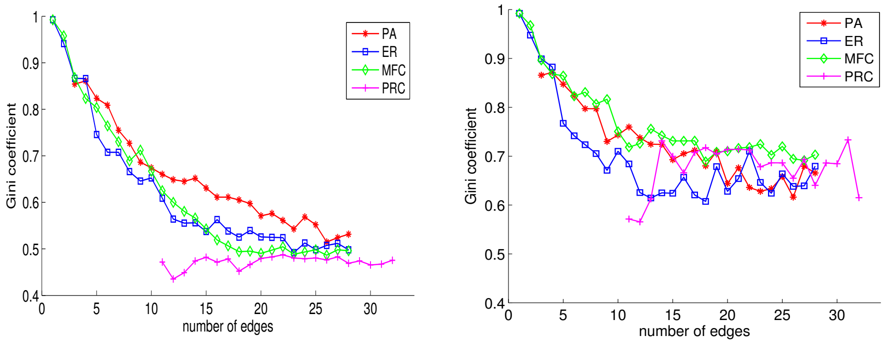

Fig. 6. Gini coefficient of given structure models with increasing number of edges. The left-hand side figure is with baseline rewards and the right-hand side one is with alternate rewards.

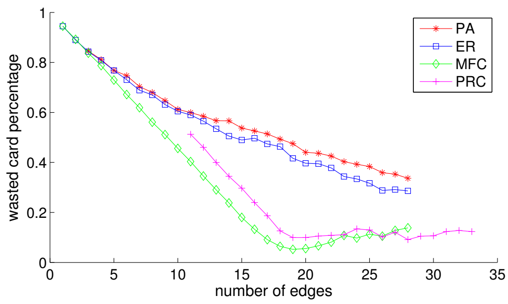

Fig. 7. Wasted card percentage with the increasing graph size.

after about 18 links. For the alternate reward structure, all the algorithms are in the same equivalence class. This is because once a few green-green links are formed, it is difficult to change the inequality of the world.

We then investigated the number of wasted cards in equilibrium, i.e., the number of cards that did not yield any reward to the player holding it. Fig. 7 shows the number of wasted cards as a percentage of the total number of cards in a society. We see that wasted cards explains a lot of the phenomena in social welfare. The MFC and PRC algorithm, which has an MFC component, waste the fewest cards because that is part of their process. The others form links that are not as useful in allowing players to use their cards. ER performs slightly better that PA because it does not overload particular users with large numbers of links. Thus as fewer cards are wasted, social welfare improves. This similarly explains the Gini coefficient because as more cards are used, we have fewer users with low or no rewards. Nevertheless, it is interesting to note that while ER wastes more cards than MFC and PRC, it does not perform worse in terms of

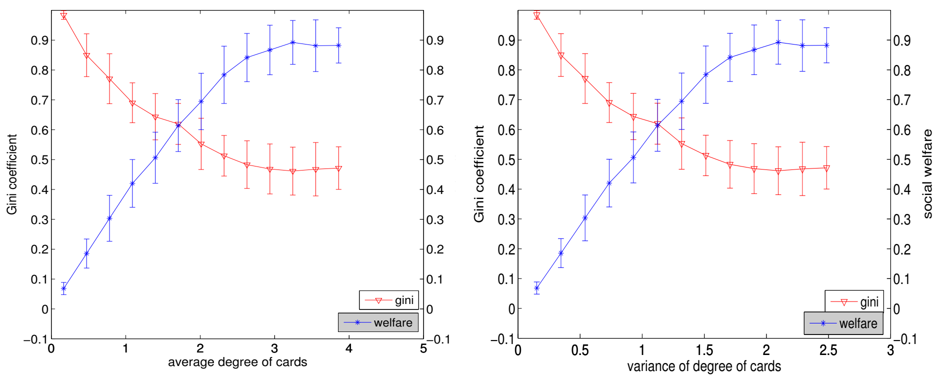

Fig. 8. Social welfare and Gini coefficient by average and variance of degree in graph with baseline rewards.

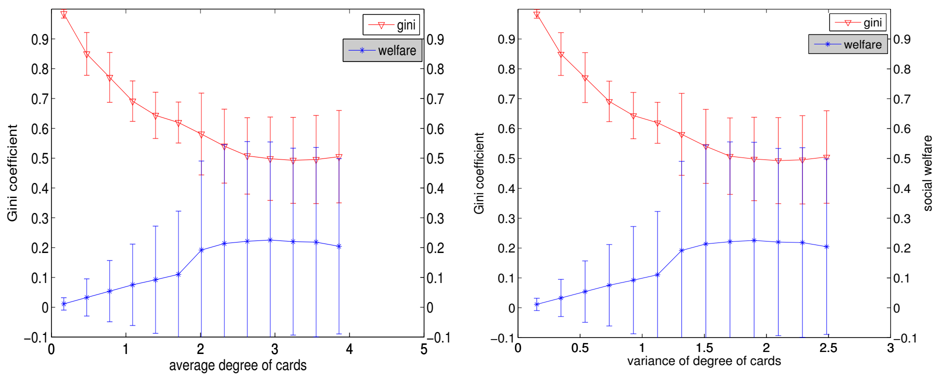

Fig. 9. Social welfare and Gini coefficient by average and variance of degree in graph with alternate rewards.

inequality. This remains an open question. Interestingly, with half the possible links (33), we still have about 10% of cards being wasted.

We also looked at the impact of network properties on outcomes. Fig. 8 shows social welfare and Gini coefficient as a function of the average and variance of the degrees of the nodes in the graph. Clearly, this will depend on the card and reward structure. In our case, both average and variance of degree showed similar curves in increasing social welfare and decreasing inequality. The inequality curves are similar in both reward structures and the social welfare curves are close to the best performing algorithms as a function of graph size. We believe the Networked Resource Game is a good starting point for modeling and investigating the complexities and design of economies of resource-bounded and socially networked agents.

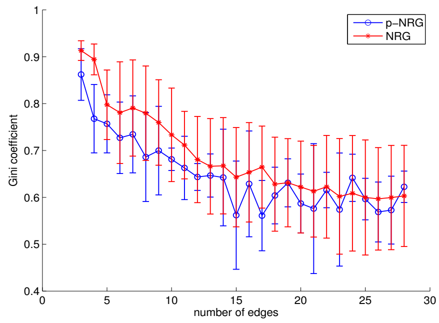

Fig. 10. Gini coefficient comparison between NRG and p-NRG in the ER model.

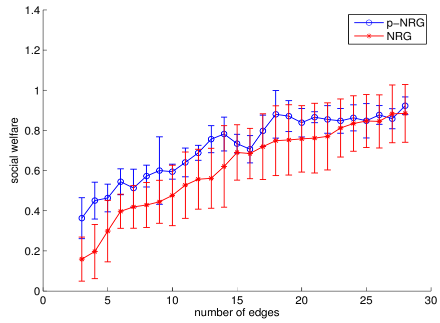

Fig. 11. Social welfare comparison between NRG and p-NRG in the ER model.

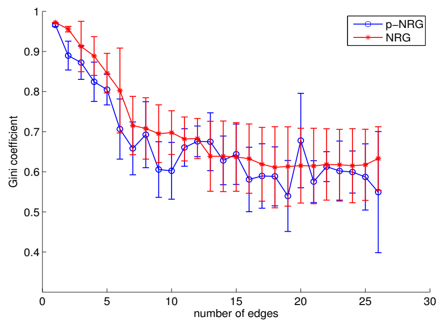

Fig. 12. Gini coefficient comparison between NRG and p-NRG in the MFC model.

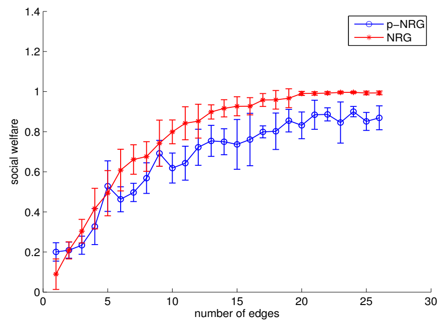

Fig. 13. Social welfare comparison between NRG and p-NRG in the MFC model.

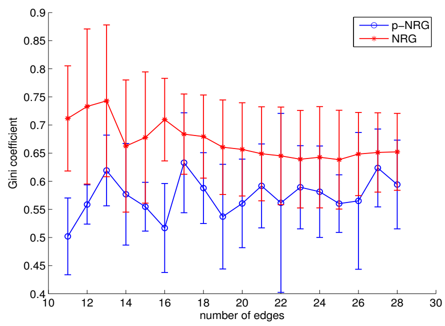

Fig. 14. Gini coefficient comparison between NRG and p-NRG in the PRC model.

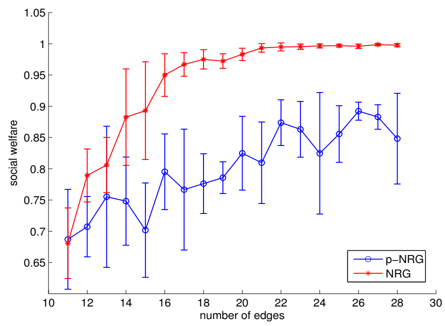

Fig. 15. Social welfare comparison between NRG and p-NRG in the PRC model.

6 Positive Feedback

In this section, we introduce a new model that we can use to simulate the both cooperations and competitions in a game. In each game, we like to reassign the cards to players based on his previous rewards. This is like that the rich people are likely to get more resources while such resources may help him become richer. This phenomenon is modeled be the following the following NRG model with positive feedback (p-NRG).

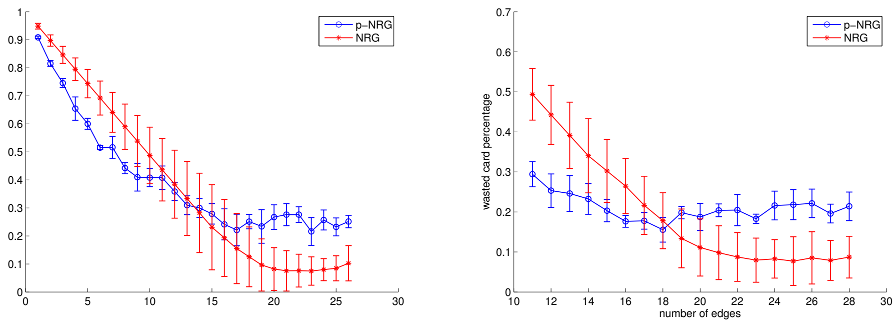

Fig. 16. Wasted card percentage, comparison of the baseline and positive-feedback NRG model given with the ER (left) and MFC (right) structure.

Given a game structure (cards, rewards and a graph), each game is conducted as a sequence of sub-games. In a sub-game, the agents only play their cards once. Like in the classical NRG, each player is first given a number of cards chosen uniformly from 1 to 5:

$$|C_i| \sim U(1, 5)$$

We calculated each player's (p_i) welfare: W_i at the end of each sub-game. At the beginning of a sub-game, each player will be re-assigned with new cards provided by a *card-pool* wit infinite number of cards. At each round of the sub-game, N cards will be drawn from the pool based on a given probability distribution on card types. For a particular player p_i, the number of new cards he can get is depending on the welfare W_i he obtained at this round. In other others, the ones with large welfare values tends to get more new cards at the next round of sub-game. The probability for getting more new cards is calculated by

$$P_i = \frac{W_i + \theta}{\sum_{i=1}^{N} W_i + N\theta} \tag{8}$$

where $\theta > 0$ is smoothing factor used to avoid getting zero probabilities. In the following experiments, we first assign the card-pool distribution by:

$$P([\text{green red yellow}]) = [1/6\ 2/6\ 3/6]$$

Each game is consisting 5 sequential sub-games. In this new game, the Gini coefficient of each game is the average value of the five sub-games, and social welfare is the sum of five sub-games.

The experimental results are presented in Fig. 10 to 17. In comparisons of NRG with and without positive feedback, we found that the results are similar for the ER and MFC models in both social welfare and Gini coefficient. However, we found there is a significant variance in the experiments with the PRC model. As we can see from Fig. 14 and 15, the p-NRG model is less than the classical NRG model in both the social welfare value and Gini coefficient. By introducing the positive feedback, the system becomes less productive in terms of social welfare. It hurts the economy somehow by introducing such circular causations. However, the social inequality is roughly the same comparing to the classical NRG. Based on our observation, positive feedback may hurt social welfare but won't influence the social inequality significantly.

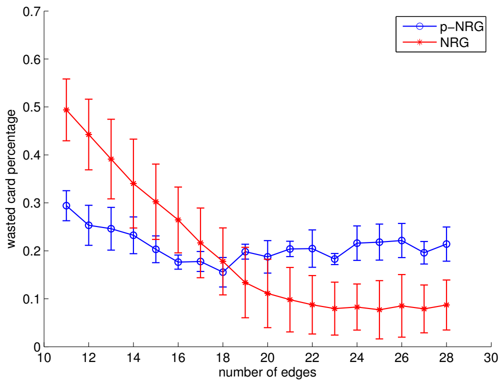

Fig. 17. Wasted card percentage, comparison of the baseline and positive-feedback NRG model given with the PRC structure.

7 Conclusions and Future Work

In this chapter, we proposed a graphical game which is referred to as the Networked Resource Game. Based on this game, we investigated how network structure may influence the social welfare and inequality measured by the Gini coefficient. Based on empirical evaluations, we found that different network structure may lead to different game dynamics in terms of increasing social welfare and decreasing Gini Coefficient. Efficient interactions of players will increase both the social welfare as well as social equality. For the NRG with positive feedback, we found that introducing of positive feedback may lead to a less productive society but it won't cause significant social inequality comparing to the classical NRG model.

One potential future direction is using these properties as part of the network formation process because they may be more easily estimated than the requirements of the processes we presented. We also plan on investigating games where more than two players can collaborate. It is also a challenge to investigate appropriate outcomes for graphs as the scale of the society grows as equilibrium discovery will become more computationally demanding.

Acknowledgements

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